Let X be a random variable with $E(X^n)=0.8$, $n=1,2,3\cdots\infty$. In the following probide complete derivation and explanations of results.

a) Find the moment generating ~~function~~ function of X.

b) Find the Characteristic Function of $Y=(-1)^X$

Solution:

a) $M_X(t)=E(e^{tX})$, is the moment generating function of X. by talor Poly~~nor~~mial.

$$M_X(t)=M_X(0)+\frac{M_X'(0)}{1!}t+\frac{M''(0)}{2!}t^2+\cdots+\frac{M^{(n)}(0)}{n!}t^n+\cdots$$

$$=\overset{E(X^0)}{\cancel{M_X(0)}}+\frac{E(X)}{1!}t+\frac{E(X^2)}{2!}t^2+\cdots+\frac{E(X^n)}{n!}t^n+\cdots$$

$$=E(1)+\frac{0.8}{1}t+\frac{0.8t^2}{2!}+\cdots+\frac{0.8}{n!}t^n+\cdots$$

$$=1+0.8\left(t+\frac{t^2}{2!}+\cdots+\frac{t^n}{n!}+\cdots+1-1\right)$$

$$=1+0.8(e^t-1)=0.2+0.8e^t$$

b) $\Phi_Y(w)=E(e^{jwY})=E(e^{jw(-1)^X})$

by $M_X(t)=0.2+0.8e^t$, we know $X\sim$ Binomial (0.8), the

X	0	1
P	0.2	0.8

$$\Phi_Y(w)=E(e^{jw(-1)^X})=e^{jw(-1)^0}\times0.2+e^{jw(-1)^1}\times0.8$$

$$=0.2e^{jw}+0.8e^{-jw}$$