

**Q:** A pair of dice is rolled until a sum of either 5 or 7 appears. Find the probability that a 5 occurs first.

**A:** Let  $E_n$  denote the event that a 5 occurs on the  $n$ th roll and no 5 or 7 occurs on the first  $n-1$  rolls.

Sum	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$P(X+Y=5) = \frac{4}{36} = \frac{1}{9}$$

$$P(X+Y=7) = \frac{6}{36} = \frac{1}{6}$$

$$P(X+Y=5 \text{ or } X+Y=7) = \frac{4+6}{36} = \frac{10}{36} = \frac{5}{18}$$

$$P(E_n) = \left(1 - \frac{5}{18}\right)^{n-1} \frac{1}{9}$$

$$\sum_{n=1}^{\infty} P(E_n) = \sum_{n=1}^{\infty} \left(1 - \frac{5}{18}\right)^{n-1} \frac{1}{9} = \frac{1}{9} \sum_{n=1}^{\infty} \left(\frac{13}{18}\right)^{n-1}$$

$$= \frac{1}{9} \left(1 + \frac{13}{18} + \left(\frac{13}{18}\right)^2 + \left(\frac{13}{18}\right)^3 + \dots\right) = \frac{1}{9} \cdot \frac{1}{1 - \frac{13}{18}} = \frac{18}{9} = \frac{2}{1}$$

$$= \frac{2}{5}$$

∴ the probability that a 5 occurs first is  $\frac{2}{5}$