求解方程组 $\begin{cases} \dfrac{dx}{dt} = p(t)x + q(t)y & ① \\ \dfrac{dy}{dt} = q(t)x + p(t)y & ② \end{cases}$ 其中$p(t)$, $q(t)$在$[a,b]$连续.

解：①+② $\dfrac{dx}{dt} + \dfrac{dy}{dt} = [p(t)+q(t)]x + [p(t)+q(t)]y$

$\dfrac{d(x+y)}{dt} = [p(t)+q(t)][x+y]$, $\dfrac{1}{x+y}d(x+y) = [p(t)+q(t)]dt$

$\ln|x+y| = \int p(t)+q(t)dt + C_0$, $x+y = C_1 e^{\int p(t)+q(t)dt}$

$$x+y = C_1 e^{\int p(t)+q(t)dt} \quad ③$$

①−② $\dfrac{dx}{dt} - \dfrac{dy}{dt} = [p(t)-q(t)]x - [p(t)-q(t)]y$

$\dfrac{d(x-y)}{dt} = [p(t)-q(t)](x-y)$, $\dfrac{1}{x-y}d(x-y) = [p(t)-q(t)]dt$

$\int \dfrac{1}{x-y}d(x-y) = \int p(t)-q(t)dt$, $x-y = C_2 e^{\int p(t)-q(t)dt}$ ④

③+④ $2x = C_1 e^{\int p(t)+q(t)dt} + C_2 e^{\int p(t)-q(t)dt}$

$$x = \frac{1}{2}[C_1 e^{\int p(t)+q(t)dt} + C_2 e^{\int p(t)-q(t)dt}]$$

③−④ $2y = C_1 e^{\int p(t)+q(t)dt} - C_2 e^{\int p(t)-q(t)dt}$

$$y = \frac{1}{2}[C_1 e^{\int p(t)-q(t)dt} - C_2 e^{\int p(t)-q(t)dt}]$$

所求微分方程组的通解为

$$\begin{cases} x = \frac{1}{2}[C_1 e^{\int p(t)+q(t)dt} + C_2 e^{\int p(t)-q(t)dt}] \\ y = \frac{1}{2}[C_1 e^{\int p(t)+q(t)dt} - C_2 e^{\int p(t)-q(t)dt}] \end{cases} \quad t \in [a,b]$$